HALE SCHOOL

WA Exams Practice Paper D, 2015

Question/Answer Booklet

MATHEMATICS METHODS UNIT 1

Section Two: Calculator-assumed

SOLUTIONS

Student Number: In figures | | | | | | | | |

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	150	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

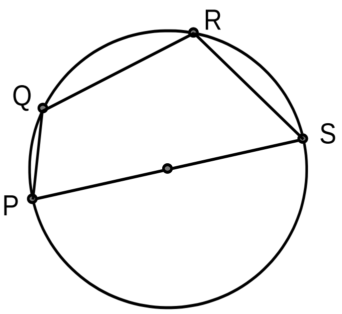
Section Two: Calculator-assumed **(98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 9 **(7 marks)**

P, Q, R and S are points that touch the circumference of a circle with diameter 22 cm. PS is a diameter of the circle, PQ has length 6 cm and RS has length 11 cm.



(a) Determine the area of the minor segment bound by the chord RS. (2 marks)

$$A=\frac{1}{2}(11)^2\left(\frac{\pi}{3}-\sin\frac{\pi}{3}\right)$$
$$=10.961 \text{ cm}^2$$

(b) Determine the length of the minor arc PQ. (3 marks)

cosine rule to find angle QOP:

$$\theta=\cos^{-1}\left(\frac{11^2+11^2-6^2}{2(11)(11)}\right)$$
$$\theta=0.55245^r$$
$$L=r\theta$$
$$=11(0.55245)$$
$$=6.077 \text{ cm}$$

(c) Find the area of quadrilateral PQRS. (2 marks)

$$A_1=\frac{1}{2}\times 11\times 11\times\sin(\pi/3)$$
$$A_2=\frac{1}{2}\times 11\times 11\times\sin(0.55245)$$
$$A_3=\frac{1}{2}\times 11\times 11\times\sin(\pi-\pi/3-0.55245)$$
$$A=52.396+31.749+60.475=144.62 \text{ cm}^2$$

Question 10 **(17 marks)**

(a) The graph of $y = a\,b^x$ passes through the points $(1,1)$ and $(2,5)$. Find the values of a and b. (3 marks)

$$\boxed{a = 0.2 \text{ and } b = 5}$$

(b) Simplify, leaving your answer *factorised in index form*: (1 mark)

$(5x^2 + 1)\sqrt[3]{5x^2 + 1}$

$$\boxed{(5x^2 + 1)^{4/3}}$$

(c) Express $450\ 780\ 200$ in scientific notation to 3 significant figures. (2 marks)

4.51×10^8

(d) Describe the transformations required to change the graph of $y = 3^x$ into the graph of $y = -3^{x+5} + 7$. (4 marks)

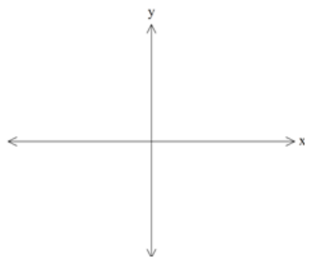
Order:

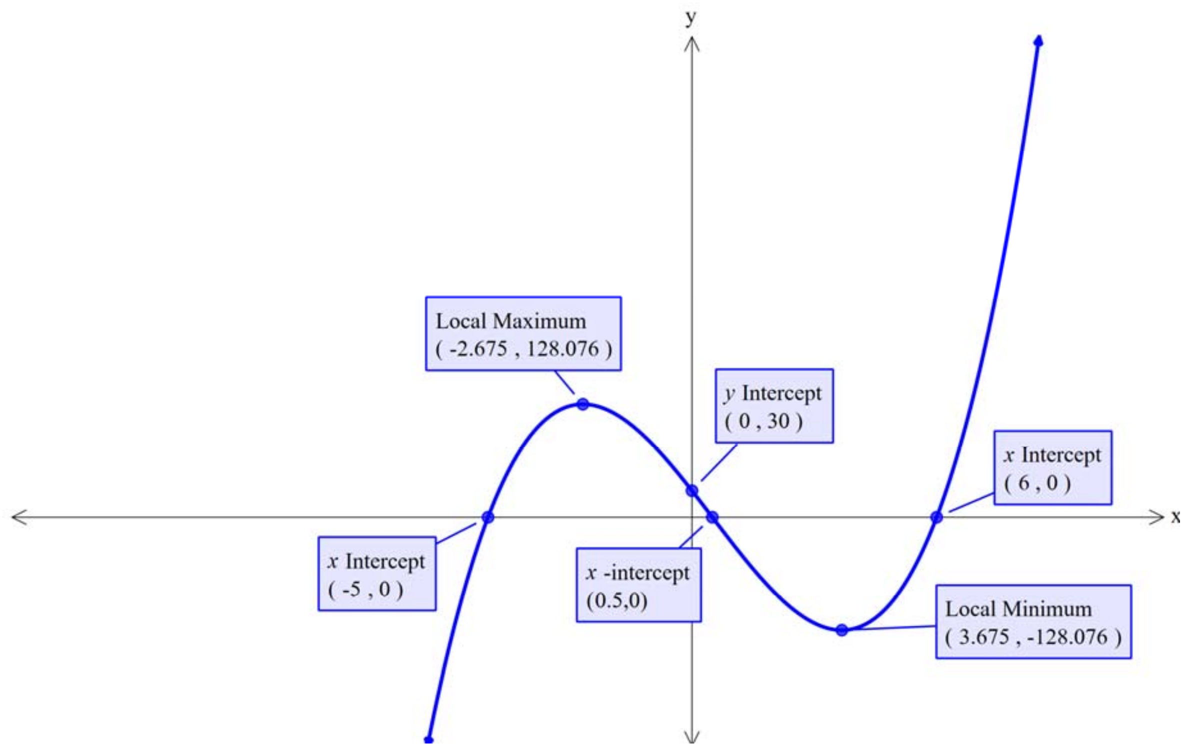
Translated 5 units left

Reflected in the x-axis

Translated 7 units up

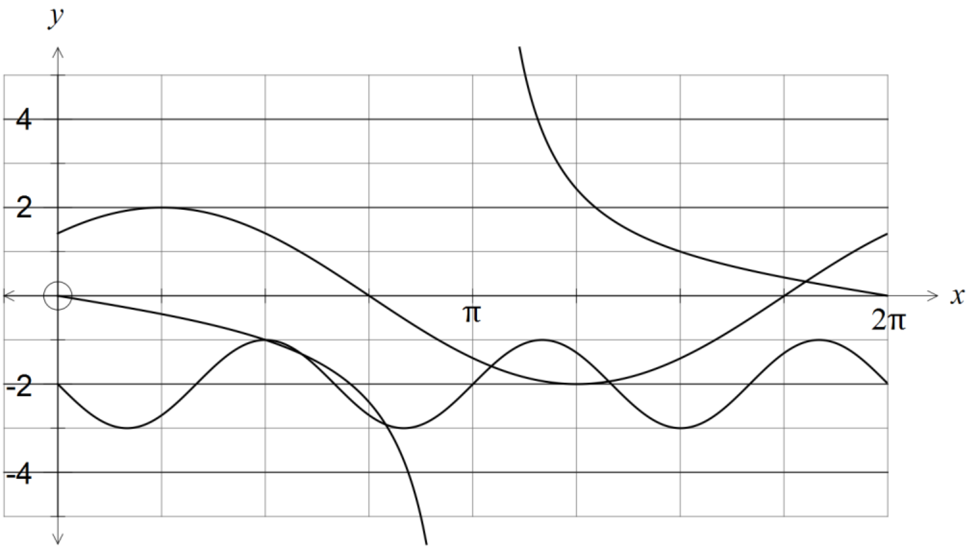
(e) Sketch the function $y = 2x^3 - 3x^2 - 59x + 30$ on the axes below, labelling all intercepts and turning points. (7 marks)





Question 11 **(6 marks)**

The graphs of the functions $y = a\cos(x+b)$, $y = \sin(cx) + d$, $y = e\tan(fx)$ are shown below, where a, b, c, d, e and f are real constants.



State the values of constants a, b, c, d, e and f .

$a = 2$

$b = -\dfrac{\pi}{4}$

$c = -3$

$d = -2$

$e = -1$

$f = 0.5$

(alternative: $e = 1$, $f = -0.5$)

Question 12 **(6 marks)**

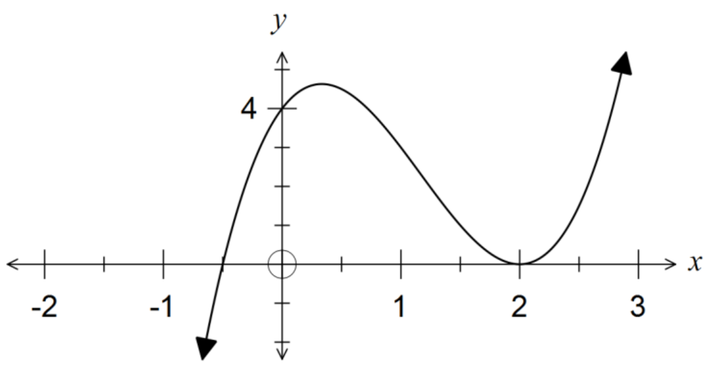
(a) The graph of $y = x^2 + px + q$ has a turning point at (3, -2). Determine the coordinates of the y-intercept of the parabola. (2 marks)

$$y = (x-3)^2 - 2$$

$$y = (-3)^2 - 2$$
$$= 7$$

$$(0,\ 7)$$

(b) The graph of $y = ax^3 + bx^2 + cx + d$ is shown below.



Determine the values of $a,\ b,\ c$ and d. (4 marks)

$$y = a(x+0.5)(x-2)^2$$

$$4 = a(0.5)(4) \ \Rightarrow\ a = 2$$

$$y = 2(x+0.5)(x-2)^2$$
$$= 2x^3 - 7x^2 + 4x + 4$$

$$a = 2,\ b = -7,\ c = 4,\ d = 4$$

Question 13 **(6 marks)**

In the year 2000, in a game park in Africa it was estimated that there were approximately 700 wildebeest and that their population was increasing at 3% per year. At the same time, in the park there were approximately 1850 zebras and their population was decreasing at the rate of 4% per year.

(a) Write two separate equations, one for the number of zebra t years after the year 2000 and one for the number of wildebeest t years after the year 2000. (4 marks)

$$w = 700\,(1.03)^t$$

$$z = 1850\,(0.96)^t$$

(b) After how many years was the number of wildebeest greater than the number of zebras? (2 marks)

$\boxed{13.81 \text{ years}}$

Question 14 **(7 marks)**

(a) A parabola with vertex at (2, -1) passes through the points (3, 0) and (3, -2).

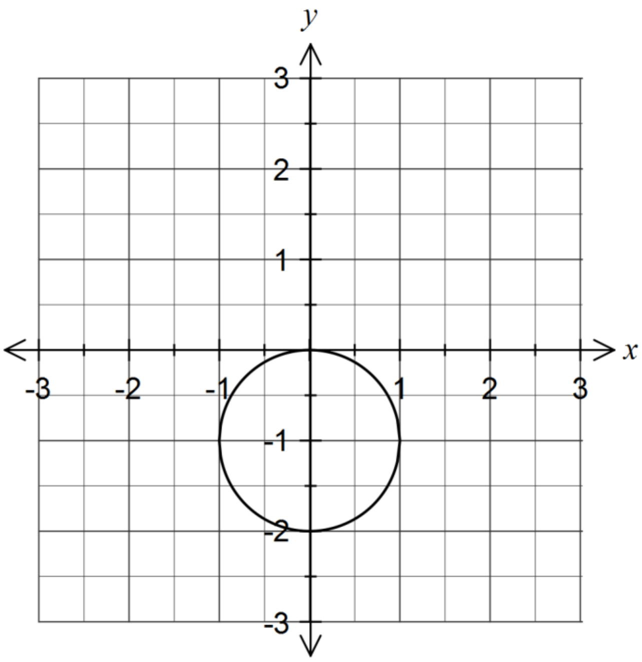
(i) State the equation of the axis of symmetry for this parabola. (1 mark)

$$\boxed{y = -1}$$

(ii) Determine the equation of the parabola. (2 marks)

$$\boxed{(y+1)^2 = x - 2}$$

(b) Sketch the graph of $x^2 + (y+1)^2 = 1$. (2 marks)



(c) Show that the circle with equation $x^2 + y^2 - 22x + 30y + 310 = 0$ has a radius of 6.
(2 marks)

$$(x-11)^2 + (y+15)^2 = -310 + 11^2 + 15^2$$
$$(x-11)^2 + (y+15)^2 = 36$$
$$r = 6$$

Question 15 **(6 marks)**

(a) Determine the smallest positive solution to the equation $3\cos(3x) = 1$, where x is in degrees, giving your solution to three significant figures. (2 marks)

$\boxed{23.5^\circ}$

(b) Show that $\cos(2\theta) = \cos^2\theta - \sin^2\theta$. (2 marks)

$$\begin{aligned}\cos(2\theta) &= \cos(\theta + \theta)\\ &= \cos\theta \times \cos\theta - \sin\theta \times \sin\theta\\ &= \cos^2\theta - \sin^2\theta\end{aligned}$$

(c) Show that the exact value of $\tan 105^\circ$ is $-\left(\sqrt{3} + 2\right)$. (2 marks)

$$\begin{aligned}\tan(105) &= \tan(45 + 60)\\ &= \frac{\tan(45) + \tan(60)}{1 - \tan(45)\tan(60)}\\ &= \frac{1 + \sqrt{3}}{1 - \sqrt{3}}\\ &= -\sqrt{3} - 2\\ &= -\left(\sqrt{3} + 2\right)\end{aligned}$$

Question 16 **(7 marks)**

In triangle ABC, $a = 15.4$ cm, $b = 12.8$ cm and $\angle B = 50°$.

(a) If ABC is an **acute-angled** triangle,

(i) write down an equation that could be solved to determine the size of $\angle A$. (1 mark)

$$\boxed{\frac{15.4}{\sin A} = \frac{12.8}{\sin 50}}$$

(ii) determine the size of $\angle A$. (1 mark)

$$\boxed{A = 67.2°}$$

(b) If ABC is an **obtuse-angled** triangle,

(i) write down an equation that could be solved to determine the length c. (2 marks)

$\angle A = 180 - 67.2 = 112.8° \Rightarrow \angle C = 180 - 112.8 - 50 = 17.2°$

$\frac{c}{\sin 17.2} = \frac{12.8}{\sin 50}$ or $12.8^2 = c^2 + 15.4^2 - 2c(15.4)\cos 50$

(ii) determine the length c. (1 mark)

$\boxed{c = 4.93 \text{ cm}}$

(c) Determine the largest possible area of triangle ABC. (2 marks)

Largest if triangle ABC is acute angled.

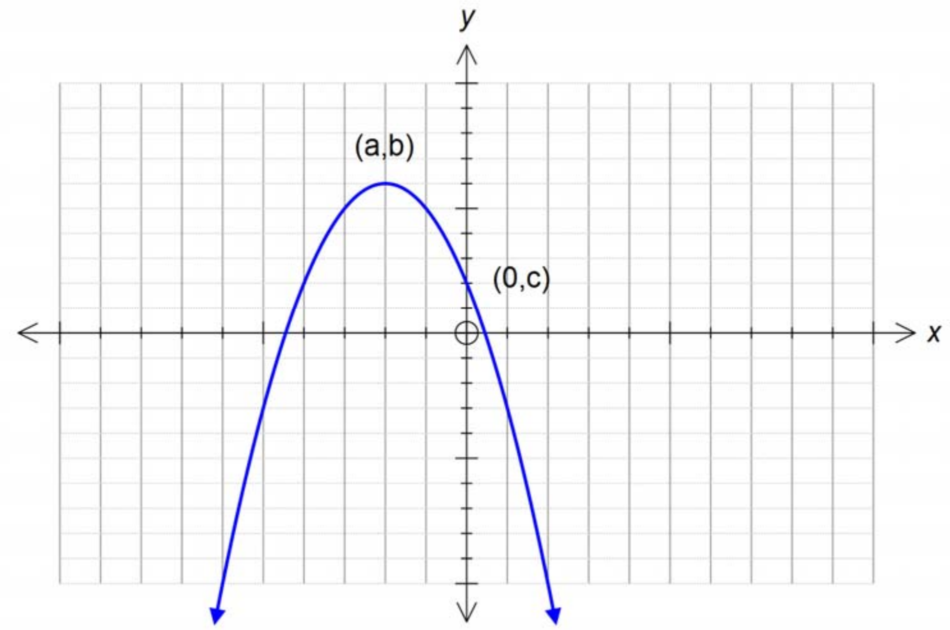
$\angle C = 180 - 67.2 - 50 = 62.8°$

Area $= 0.5(15.4)(12.8)\sin 62.8$

$= 87.7$ cm^2

Question 17 **(6 marks)**

Consider the function $g(x)$ below, with turning point (a, b) and y intercept $(0, c)$.



In terms of a, b and c:

(a) Determine the coordinates of the turning point for $f(x)$, if $f(x) = g(x - 3) + 2$ (2 marks)

T.P. $(a+3, b+2)$ ✓✓

(b) Determine the coordinates of the y intercept for $p(x)$, if $p(x) = -g(-x) + 3$ (2 marks)

$(0, -c+3)$ ✓✓

(c) State the domain and range of $m(x)$, if $m(x) = -g(x-1)$. (2 marks)

D: All R ✓

R: $y \geq -b$ ✓

6

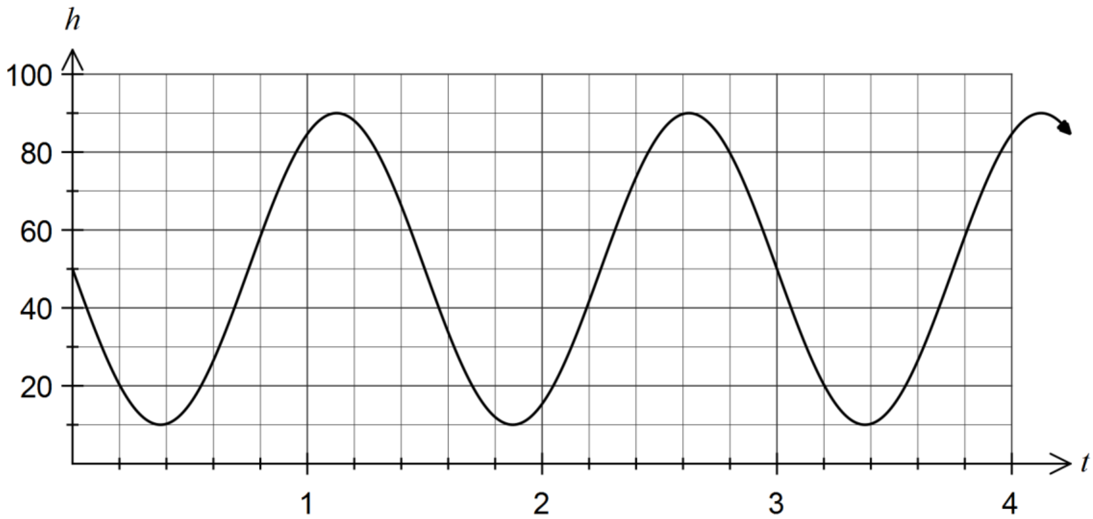
Question 18 **(9 marks)**

The height above the ground, h cm, of a small weight attached to a spring was observed to vary according to the function $h(t) = 50 - 40\sin\left(\frac{4\pi t}{3}\right)$, where t is the time in seconds since measurements began.

(a) Determine the initial height of the weight above the ground. (1 mark)

50 cm

(b) Sketch the graph of $h(t)$ on the axes below for $0 \le t \le 4$. (3 marks)



(c) What is the period of the motion? (1 mark)

$2\pi \div \frac{4\pi}{3} = 1.5$ seconds

(d) How far did the weight travel during the first six seconds? (2 marks)

Travels 4 complete cycles in 6 s.
Travels 160 cm each cycle.

Travels $4 \times 160 = 640$ cm

(e) For how long during the first six seconds was the weight within 20 cm of the ground? (2 marks)

Solve $20 = 50 - 40\sin\left(\frac{4\pi t}{3}\right)$ to get solutions of $t = 0.202$ and $t = 0.548$.

$0.548 - 0.202 = 0.346$ in 1.5 s, so $4 \times 0.346 = 1.38$ seconds in 6 s.

Question 19 **(6 marks)**

The number of a certain species of fish, in the North Atlantic Ocean, can be modelled by the function:

$$n(t) = 12\,(1.005)^t$$

Where t is the number of years and $n(t)$ is the number in thousands.

Question 23 **(6 marks)**

The number of a certain species of fish, in the North Atlantic Ocean, can be modelled by the function:

$$n(t) = 12(1.005)^t$$

Where t is the number of years and $n(t)$ is the number in thousands.

(a) What is the % increase in fish each year? (1 mark)

$\frac{1}{2}\%$ ✓

(b) What is the fish population in 5 years? (1 mark)

12.303 Thousand

≈ 12303 ✓

(c) After how many years will the number of fish reach 30 thousand? (2 mark)

$30 = 12(1.005)^t$ ✓

$t \approx 183.71$ YRS

≈ 184 YRS ✓

Another population of fish can be modelled by:

$$m(t) = 8(1.035)^t$$

Where t is the number of years and $m(t)$ is the number in thousands.

(c) Determine after how many years that the second species of fish will outnumber the first species. (2 marks)

$12(1.005)^t = 8(1.035)^t$

$t \approx 13.78$ YRS ✓

FROM YR 14 ON ✓

6

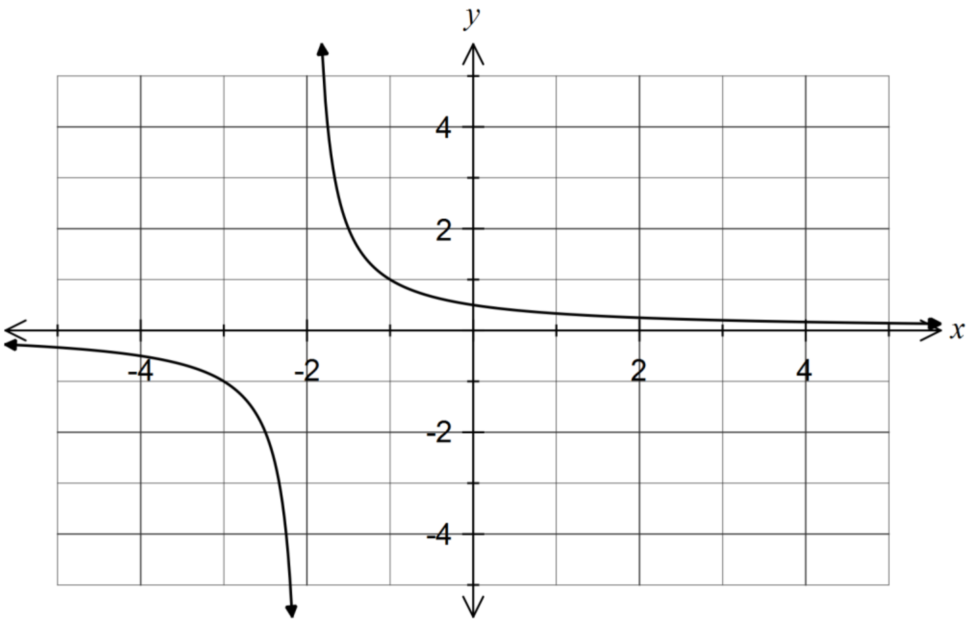
Question 20 **(8 marks)**

The function f is defined by $f(x) = \dfrac{1}{x+2}$.

(a) State the domain and range of $f(x)$. (2 marks)

$\{x : x \in \mathbb{R},\ x \neq -2\}$ and $\{y : y \in \mathbb{R},\ y \neq 0\}$

(b) Sketch the graph of $y = f(x)$. (2 marks)



(c) State the equations of the asymptotes of the graph of $y = f(x) + 3$. (2 marks)

$x = -2$ and $y = 3$

(d) Describe how to transform the graph of $y = f(x)$ to obtain the graph of

(i) $y = f(4x)$. (1 mark)

Dilate the graph horizontally by a scale factor of $\frac{1}{4}$.

(ii) $y = \dfrac{1}{x-1}$. (1 mark)

Translate the graph 3 units to the right.

Question 21 **(7 marks)**

(a) Use an algebraic method to determine the point of intersection of $y = \dfrac{2}{x-1}$ and $y = \dfrac{1}{2x+1}$. (3 marks)

$$\frac{2}{x-1} = \frac{1}{2x+1}$$
$$4x + 2 = x - 1$$
$$3x = -3$$
$$x = -1$$
$$y = \frac{2}{-1-1} = -1$$

At the point $(-1, \ -1)$

(b) The quantity P is directly proportional to another quantity h and when $h = 3.5$, $P = 1225$.

Determine the value of h when $P = 7000$. (2 marks)

$$\frac{h}{7000} = \frac{3.5}{1225}$$
$$h = 20$$

(c) The quantity C is inversely proportional to t. It is known that $C = 30$ when $t = 24$.

Determine the value of C when $t = 7.2$. (2 marks)

$$30 \times 24 = C \times 7.2$$
$$C = 100$$

End of questions

Additional working space

Question number: _________

Additional working space

Question number: _________